Problem C

Your game development studio, Ad Hoc Entertainment, is currently working on a billiards-based app they’re calling Pool Shark. Players face a sequence of increasingly devious pool puzzles in which they need to carefully position and aim a single billiards shot to sink multiple pool balls.

You’ve just done the first round of user testing and the feedback is terrible — players complain that the physics of your pool game is neither fun nor intuitive. After digging into it, you realize that the problem isn’t that your physics code is bad, but rather that most people just don’t have much intuition about how physics works. Fortunately, no one requires your physics to be realistic. After this liberating realization, your team experiments with a few models, eventually settling on the following rule for how to resolve pool-ball collisions:

When a moving pool ball B hits a stationary ball A, A begins moving in the direction given by the vector from the center of B to the center of A at the time of the collision. Ball B’s new velocity vector is B’s original vector reflected across A’s new vector (Figure H.1). Note that A’s resulting vector is what real physics predicts, but B’s is not (unless A is glued to the table or has infinite mass). For the purposes of this problem, the speed at which the balls move is irrelevant.

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| \includegraphics[width=0.45\linewidth ]{trickshot1} |  | \includegraphics[width=0.45\linewidth ]{trickshot2} |
| Figure H.1 |  | Figure H.2 |

This actually allows for more interesting challenges, but requires new code to determine whether a particular level is feasible. You’ve been tasked with solving a very particular case:

Three balls labelled 11, 22, and 33 are placed on a table with width ww and length ll (Figure H.2). The player must place the cue ball somewhere on a dashed line lying hh units above the bottom edge of the table. The goal is to pick a distance dd from the left side, and an angle θθ such that when the cue ball is shot, the following events happen:

* The cue ball strikes ball 11, and then ricochets into ball 22, sinking ball 22 in the top left hole.
* Ball 11, having been struck by the cue ball, hits ball 33, sinking ball 33 in the top right hole.

For simplicity, assume that sinking a ball requires the center of the ball to pass directly over the center of the hole. Further assume that the table has no sides — a ball that goes out of the ww-by-ll region simply falls into a digital abyss — and thus you don’t need to worry about balls colliding with the table itself.

You need to write a program that, given values for ww, ll, hh, the position of balls 11–33, and the radius rr of the balls, determines whether the trick shot is possible.

**Input**

The input begins with a line containing two positive integers ww ll, the width and length of the pool table, where w,l≤120w,l≤120. The left hole is at location (0,l)(0,l) and the right hole is at location (w,l)(w,l).

The next line will contain 8 positive integers rr x1x1 y1y1 x2x2 y2y2 x3x3 y3y3 hh, where r≤5r≤5 is the radius of all the balls (including the cue ball), xixi yiyi is the location of ball i,1≤i≤3i,1≤i≤3, and hh is the distance the dashed line is from the front of the pool table (see the figure above, where r≤h≤(1/2)lr≤h≤(1/2)l). No two balls will ever overlap, though they may touch at a point, and all balls will lie between the dashed line and the back of the table. All balls will lie completely on the table, and the cue ball must also lie completely on the table (otherwise the shot is impossible).

**Output**

For each test case, display the distance dd to place the ball on the dashed line and the angle θθ to shoot the ball, or the word “impossible” if the trick shot cannot be done. Output θθ in degrees, and round both dd and θθ to the nearest hundredth. Always show two digits after the decimal point, even if the digits are zero.

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| **Sample Input 1** | **Sample Output 1** |
| 20 30  2 10 20 2 24 18 28 10 | 12.74 127.83 |

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| **Sample Input 2** | **Sample Output 2** |
| 20 30  2 15 20 2 24 18 28 10 | impossible |